Chapter 6

Mill's Logic as the Basis for Computer-based Cognitive Aid

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Abstract: An interpretation of Quasi Axiomatic Theory and Mill's logic is made in support of an implementation of situational logic. Mill's logic[1] is considered incomplete by most scholars and was never formalized, by Mill, to the degree that one finds in other areas of mathematical logic, such a fuzzy logics or rough sets, or the foundations of computing. Quasi Axiomatic Theory [2] builds on the originally incomplete Mill's logic and certain interpretations of the logic formalism of C S Peirce. One way to regard QAT is to think about an open system of observation where facts are accumulated from direct experience. Each of these facts is taken as being "true" because they are carefully acquired through direct observation of physical reality. The methodology for QAT is to gather observed facts, and from this set of observed facts attempt to generate a minimal set of axioms and postulates along with inference rules that may be used mechanically to assert the observed facts as well as a set of inferred assertions. The "self evidence" is derived through an empirical methodology followed by the use of specific formal reasoning. Mill's type formal reasoning has five aspects, called "logical cannons", by Mill. If this methodology is followed the set of axioms and postulates, when equipped with that set of inference rules, may be considered a situational logic. This consideration does force an analysis the nature of reification of ontological universals from the situational analyses of the particulars of situations. Particularly as applied to the first three cannons, a voting procedure is introduced; in this paper and by Prueitt in 1997[3]. This easy computation operationalized the Mill's logic as part of a general method for developing operational ontology with situational inference. This situational logic might be "plausibly" applied to generating conjectured facts that were not observed first hand.

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[1] Mill J S, *System of Logic* (1843) http://www.bartleby.com/224/0108.html

[2] Finn, Victor (1991). Plausible Inferences and Reliable Reasoning. Journal of Soviet Mathematics, Plenum Publ. Cor. Vol. 56, N1 pp. 2201-2248

[3] Prueitt, P. (1998). An Interpretation of the Logic of J. S. Mill, in IEEE Joint Conference on the Science and Technology of Intelligent Systems, Sept. 1998, NIST.

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Introduction

The means through which humans instrument cognition is an exciting and complex area of scholarship. Views on the nature of cognition and perception arise based on different assumptions about human nature. However, only slowly has this scholarship turned to behavioral and cognitive neuroscience and the many new methods used to study of brain function. The methods supporting empirical investigation heralded by Francis Bacon, Newtonian and then Mills; may have developed along with a spoken view regarding the nature of natural law. This view tacitly asserts that ALL natural phenomena, including the full spectrum of the natures of living systems, will eventually be explained by a single coherent theory. This theory is to be spoken in the language of Hilbert mathematics.

A rigorous formulation of neuroscience has been stimulated by success from higher mathematics. This success has been an avenue to explain causation in mechanical systems. Empiricism has hinted at a complete model of all the processes that support human cognition. The argument is developed well by a number of scholars, including Sir Roger Penrose[4] and Ilya Prigogine[5] that this search for a single coherent theory of everything is misplaced. An understanding of how this search is in error has taken centuries to frame. In the time of Newtonian, natural scientists did not know what we know today. For example, difficult open questions in mathematics are related to the discretization of dynamics from models based on ordinary or partial differential equations[6]. These open questions reveal perplexing difficulties in making full simulations of the processes involved in supporting human cognition.

Higher mathematics and rigorous logical systems, such as expert systems, have shown to be limited[7]. Many modern decision support systems depend on algorithms that are likewise limited. All the way down into modern materialist science, we see the same success and the same and the same points of failure. The mathematics of trajectories defined on a manifold has well-established limitations. This creates a deep challenge to academic disciplines such as artificial neural networks[8]. We fail to find closed form solutions, and thus we turn to numerical simulations. Even here, a number of factors impact our attempt to simulate all of the brain processes involved in supporting everyday cognition. A theory of discrete to continuum homologies is simply incomplete[9] and does not yet allow logical entailment to be passed back and forth between simulations of trajectories defined on a manifold. Logical and mechanical entailments are often aligned, but this alignment has not been found between the biology involved in human cognition and a formal model of mathematics.

There are things we know about the neuroscience. Mental induction is a cognitive

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[4] Penrose, Roger (2004) *The Road to Reality: A Complete Guide to the Laws of the Universe* (2004, ISBN 0-224-04447-8 (hardcover), ISBN 0-09-944068-7 (paperback))

[5] Prigogine, Ilya (1997). *End of Certainty*. The Free Press. ISBN 0684837056.

[6] Prueitt, Paul Stephen (1988) Some techniques in mathematical modeling of complex biological systems exhibiting learning, PHD Thesis, in Pure and Applied Mathematics, University of Texas at Arlington Press

[7] Willson, Victor Methodological limitations for the use of expert systems techniques in science education research Journal of Research in Science Teaching Volume 27, Issue 1, Article first published online: 18 AUG 2006

[8] Levine, S. S. (1991) Introduction to Neural & Cognitive Modeling, LEA

[9] Prueitt, Paul Stephen (completed as report to under contract - 12/18/2011) Discrete Homology to Axiomatic Systems. (3 pages)

process acting in a present moment based on certain perceptions and inferences. Mental induction exists in real time as part of perception. It has a temporal aspect that accounts for fundamental changes in a non-stationary "external" ontology; a model of the world. The results of induction include all natural languages, all well formed belief systems, and all pre-cognitive feelings experienced by humans. We do not have a perfect model for any of these phenomena. In comparison to models of stress on building support-beans in engineering or to models of volumes or probability distributions we have made only limited advancements in neuro-mathematics.

The problems associated to modeling, or simulating, human cognitive processes are not fully posed. And yet the attempt at understanding human thought goes back into our history. The neuroscience will tell us that we know a great deal about the behavioral neuroscience and the physiology of the brain system. We understand a great deal about cellular processes and processes occurring at the level of chemical proteins in the brain. But we do not have perfect models of cellular or molecular population interaction using systems of differential equations. Certainly the planar rotator models have not been successful in modeling logical entailment[10], see also Appendix A.

Part One: Simple Enumeration

Aristotle described an inference method called induction by simple enumeration. The method proposes that: if we have a number of uniform facts and we do not know of any contrary facts we can make a generalization about these facts. This type of induction is "weaker" than a method that would falsify a theory. However, the induction by simple enumeration may be close to how natural language forms, through use. It is conjectural on our part to suggest that Aristotle viewed natural language formation in this way, but we do make the conjecture that natural language forms in a fashion that involves categorical processes.

We have the viewpoint that cognitive categories form from an underlying physics. The physics self organizes under constraints that are produces of evolution. The nature of evolution is important to our viewpoint, but for now we must bracket the term and return to a more complete description later. What we are looking for is a set of methods that define natural category at two levels of observation. The first level is of components seen to be present in more than one instance. The analogy is of chemical atoms like helium that, at the same time, can be described as a single something, and yet occurs in greatly distributed locations. How does the category "helium" manifest with such regularity?

We wish to achieve a similar regular distribution of category within algorithmic systems[11]. These components have the nature of "universals" extracted as a categorical abstraction from the experience of multiple instances. Their distribution within a social media[12] is seen to use a principle called super distribution[13].

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[10] Prueitt, Paul Stephen (completed as report to under contract - 12/18/2011) Discrete Homology to Axiomatic Systems. (3 pages)
[11] Prueitt, Paul Stephen (2009) Articulating SOA in the cloud, http://www.soamag.com/I34/1109-4.php
[12] Prueitt, Paul Stephen (2011) *Stratification Theory as Applied to Neural Architecture enabling a Brain-like function for Social Networks* . Presented to Winter Chaos Conference of the Blueberry Brain Institute, Southern Connecticut State University, March 18-20 2011.
[13] Ryoichi Mori, Masaji Kawahara, "Superdistribution: The Concept and the Architecture". *Transactions of The Institute of Electronics, Information, and Communication Engineers*, vol. E73 #7, July 1990, pp.1133–1146.

Using Quasi Axiomatic Theories (QAT) developed by V. Finn (1991), a set of "facts" may be placed inside a deductive framework. The framework may become situationally grounded though perceptual categorization and induction, producing a reasoning system complete with deductive inference. However, the validity of such deductive algorithms depends on the validity of a class of underlying assumptions. In the case of our extension of QAT we make the assumption that universals, existing as parts of things, are composed to produce specific instances, which is what we experience. This stratification is different from Aristotelian assumptions in significant ways.

The Aristotelian assumptions are understood by considering his theory of causation. The classes of Aristotelian laws:

causation; formal, material, effective and final;

provide examples of induction reasoning about causation relationships. These laws are deep and had great utility.

For Aristotle, at least in the interpretation of some, the phenomenon of cause is related to similarities within a temporal sequence. The similarity relates elements and may become a model of states of situations. The similarity between two things can be stated as

$$< a, r, b >$$

where r is the relationship. Dis-similarity is provided a corresponding notation.

At least in how Aristotle's metaphysics was incorporated into Newtonian science, these similarities will be crisp in nature. No critical hidden entanglement between similarity classes is to be tolerated. The world is to be considered as a deterministic machine. It is this crispness and absence of entanglements that might be challenged given modern science and modern understanding of phenomenon like natural language and human consciousness. There is casual entanglement. There is also the absence of a full understanding of the factors involved in human behavior. In fact, it is conjectured that living systems have hidden causation due to intention and other phenomenon. We suggest that living system be regarded as open complex systems, and further suggest that Aristotle's logic is closed and simple.

Hidden categorical entanglement may be a sufficient reason why Aristotelian logic does not describe all causation in open complex systems. Nature is complete with examples of systems that behavior in non-logical ways. Something may make a transformation from one category into another category, as in metabolic activities where a molecular element is given a specific function by a catalytic process. A number of elements may be brought together and transformed into a whole that is not the same as the crisp sum of the parts. In addition to categorical entanglement, we must consider possible insufficiency in sampling and in description. The measurement of behaviors of a human being is an example of measurement insufficiency. The issues themselves become entangled.

Aristotelian logic has assisted us in developing a class of laws of causation by generalizing from descriptions of many possible cases of causation. The generalization is from a specific set of examples and assumes validity to the descriptions of the examples. However, the choice of examples, and the description of examples in some type of formal syllogistic language is more problematic than Aristotelian logic pre-supposes.

" Logic, in the Middle Ages, and down to the present day in teaching, meant no more than a scholastic collection of technical terms and rules of syllogistic inference. Aristotle has spoken, and it was the part of humbler men merely to repeat the lesson after him. . . . .

The first extension was the introduction of the inductive method by Bacon and Galileo – by the former in a theoretical and largely misunderstood form, by the latter in actual use in establishing the foundations of modern physics and astronomy. ... But induction, important as it is when regarded as a method of investigation, does not seem to remain when its work is done: in the final form of a perfected science, it would seem that everything ought to be deductive. If induction remains at all, . . . , it remains merely as one of the principles according to which deductions are effected. Thus the ultimate result of the introduction of the inductive method seems not the creation of a new kind of non-deductive reasoning, but rather a widening of the scope of deduction . . ." (Russell (1914))[14]

There is perhaps no real question about the universality of the laws developed using methods attributed to Aristotle. If the system under observation, for example Galileo's observation of the invariants of falling objects, is very stable, then deductive syllogisms are constructed around that set of laws which govern physics. However, in open systems, the system has fundamentally changing internal dynamics. In this case, the situation is more difficult.

The metaphysics of Aristotle does not have the richness of modern theories of causation. Even though Aristotelian logic has been applied to a range of phenomenon, his methods only work if the phenomenon is fully constrained by known universal law. This is clearly not the case with a class of phenomenon such as psychological motivation. The constraint from physics is "still there", always; but perhaps biology sees physics as a partial constraint and allowing of individual intention.

Part Two: Six Logical Canons

It is may be difficult to explain human inference in terms of the monotonic / non-monotonic logics fulcrum. This has been the main line of an approach towards unifying theories of logical entailment and theories of physical entailment. If we start from the neuroscience, we see things differently. To establish a different viewpoint is not necessarily the same as setting aside the history of science, logic or mathematics. We are suggesting that a modified approach will avoid a limitation that is now quite obvious.

To mention the limitation itself is controversial, and many people have written on this, so we will not venture in this direction. We appeal directly to common experience. The notion of truth from computable algorithms is not very clear. We see in our private experience that too many open and unresolved concerns exist, and that the results of computed reasoning is often shallow. Deep substructure is required for human cognition. A type of deep structure is provided by the architecture proposed by Prueitt[15]. This architecture has a "clopen" principle, which is used to open and close the axiomatic foundation to a formal system. Opening a formal system is compared with the dissolution of a, physical, coherent potential field measurable using EEG and other

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[14] Russell, Bertrant (1914). *Our Knowledge of the External World as a Field for Scientific Method in Philosophy*. Chicago and London: Open Court Publishing

[15] Prueitt, Paul Stephen (2012) American Education Bridge, technology and pedagogy. Accepted: The 3rd International Conference on Education, Training and Informatics: ICETI 2012 March 25th - 28th, 2012 – Orlando, Florida, USA

instrumentations.

Mental induction is a cognitive process acting in a present moment based on certain perceptions and inferences[16]. In scholarship on Tibetan philosophical traditions we find a representation of Eastern views on perception and inference.

> " In previous chapters, we have seen that both … admit only two forms of instrumental objects, namely particulars and universals. Both philosophers take the existence of only two instrumental objects as the warrant for admitting only two forms of awareness as instrumental: perception and inference[17]."

The instrumentation of human awareness is defined in various ways. Prueitt's stratification theory places particulars at one organizational scale and universals at an entirely different organizational scale. This theory is consistent with the notion that experience produces abstractions associated with field potentials and with metabolic processes in the brain system. The metabolic processes may be modeled as de-coupled planar rotators, and the field potential as the emergent field manifold as modeled by the Pribram neurowave equations [18] and by weakly coupled oscillator systems[19] [20]. The stratification hypothesis by Prueitt [21] is the bases for his modification of the use of Mill's logic. Mental induction exists in real time. Mental induction involves more than one organizational scale[22]. It has also a temporal aspect that accounts for fundamental changes in a non-stationary "external" ontology.

Quasi-axiomatic theories may provide algorithms that are suited for modeling open systems and thus for modeling the inductive processes involved in human understanding of open systems. We say "may" because the attempt at developing quasi-axiomatic systems; e.g., systems that allow axiom sets to be replaced, has not been the main focus of scholarship, research and development. In our extension of QAT, we see human reasoning as supported algorithmically through plausible reasoning and periodic updates to axiom sets. Thus our definition of quasi-axiomatics is not going to be the same as QAT was in Soviet science. Most important is the possibility that the axiomatic foundation of an entire system can be regenerated quickly when results of reasoning provide incorrect results[23].

The father of Soviet QAT is Victor Finn[24] [25]. His work is based on the work of Francis

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[16] Dunn, John (2004). Foundations of Dharmakiriti's Philosophy, Studies in Indian and Tibetan Buddhism, Wisdom Books.

[17] Ibid page 145, Beginning sentences in Chapter 3, "The Basis of Inference".

[18] Pribram, K. H. (1991). Brain and Perception: Holonomy and Structure in Figural Processing. Hillsdale, NJ: Lawrence Erlbaum Associates. See Appendix B.

[19] J. Kowalski; A. Ansari; P. Prueitt; R. Dawes and G. Gross (1988.) On Synchronization and Phase Locking in Strongly Coupled Systems of Planar Rotators. Complex Systems 2, 441-462.

[20] Prueitt, Paul Stephen (completed as report under contract - 12/18/2011) Discrete Homology to Axiomatic Systems. (3 pages)

[21] Prueitt, Paul S. (1995) A Theory of Process Compartments in Biological and Ecological Systems. In the Proceedings of IEEE Workshop on Architectures for Semiotic Modeling and Situation Analysis in Large Complex Systems; August 27-29, Monterey, Ca, USA; Organizers: J. Albus, A. Meystel, D. Pospelov, T. Reader

[22]

[23] Prueitt, Paul (2009) - "The Service Engine: Structured Communication using Modern Service Technologies" SOA Magazine, http://www.soamag.com/I30/0709-1.asp

[24] Finn, Victor (1996a). Plausible Reasoning of JSM-type for Open Domains. In the proceedings of the Workshop on Control Mechanisms for Complex Systems: Issues of Measurement and Semiotic Analysis: 8-12 Dec. 1996

Bacon[26] and J. S. Mill. All three men developed a theory of causation based on "induction by simple enumeration". At the core of a common core to these theories is a specific type of similarity analysis. For Bacon, Mill and Finn similarity defines classes of instances and facts and conjectures framed within the context of these instances. Mill gave a general analysis of the existing, mainly Aristotelian, theories of inductive proof and provided a set of formula and criteria related to the problems of scientific reasoning. More specifically, Mill formulated five "canons of reasoning" about casual hypotheses. As such there is a consistency between Mill and Aristotle.

The central issues we address in our extension of QAT is in the notion that a single logical system might be found, sufficient to reasoning about any situation. The issue is the issue of coherence[27]. We may maintain the consistency in many approaches to the problem of modeling cognition while at the same time moving away from the notion that a single reasoning system should be deemed universal in nature. This focus on real time interfaces between humans and computing systems is a core part of what we will propose.

Mill's Canons (1872) may be summarized as follows.

> First Canon [Method of Agreement]: If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon.
>
> Second Canon [Method of Difference]: If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former; the circumstance in which the two instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.
>
> Third Canon [Joint Method of Agreement and Difference]: If two or more instances in which the phenomenon occurs have only one circumstance in common, while two of more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which the two sets of instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.
>
> Fourth Canon [Method of Residues]: Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents. There is a possible matching between everything not explained and that part of our understanding that is not explaining anything.
>
> Fifth Canon [Method of Concomitant Variations]: Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation. This canon is similar to the use of dependant and independent factors in statistical studies.

Mill's motivation was to formalize a theory of inferential inductive knowledge based on the concept of natural law. For Mill, natural law referred to relationships between

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[25] Finn, Victor (1996b) Basic concepts of Quasi Axiomatic Theory, presented at the QAT Teleconference, New Mexico State University and the Army Research Office, December 13, 1996

[26] Farrington, Benjamin (1964). The Philosophy of Francis Bacon; an Essay on Its Development from 1603 to 1609,. [Liverpool]: Liverpool UP,. Print.

[27] Prueitt, Paul Stephen (Dec 24, 2011). "Technical Foundations to Stratified Theory and Articulated Machines", an internal report. 25 pages

antecedent and consequent events that are universally invariant. The validity of the inductive generalization was grounded in the invariance of these natural laws. As we will see, this is a point of disagreement between Mill and Peirce. The disagreement is in essence about whether or not there is a unique nature to each perception in real time.

In the Peircean sense, the interpretant makes a judgment about a set of signs, and in this way imparts something that is not in the signs at all. We interpret this point to build situational logics that are bi-level and thus separated except during a meta-phenomenon of emergence. The Peircean notion of, human interpretant might be more fully supported, and is recognized to have something essential that the computed reasoning system cannot have, due to the nature of computing system and the nature of natural (non-computed) systems. Perhaps this was the direction the Finn took, perhaps not. However, my work is intending to create a real time interface through which a human interpretation of formal results might immediately have an impact on the state of computed reasoning.

We take a "Peircean interpretation" of the canons by making two changes in philosophy. First, the "cause" we are looking for is a "compositional cause" where basic elements are composed into emergent wholes. Pierce used the metaphor of chemical compounds having been composed by atoms. The set of compositional causes of chemical properties is then ascribed as the presence or absence of specific atoms in the chemical composition. However, if the full set of laws of chemistry are not known, we have a method for discovering the laws of chemistry. Further, if the full set of laws of human behavior is not fully discovered, we may have a method that makes some contribution to our understanding of human behavior. Whether or not there is a full set of laws is left open.

Second, the invariance that we look for are situational invariants that are defined across basins of similarity within specific organizational contexts. The issue of context is perhaps best seen in the application of Mill's logic to text understanding.

All of the canons have a common feature: *there are descriptions of an occurrence of some phenomenon under investigation and there are related descriptions in which the phenomenon does not occur.* We take is a necessary reminder that the description may be incorrectly stated, or that the absence of a specific description may be a consequence of a measurement problem.

Based on formal means, conclusions are drawn regarding the causes of phenomenon in situational context.

The starting situation assumes that propositions of the following form are given.

"p is an observed property of object O".

The proposition is taken as an empirical observation. Finn, in his 1991 paper, would write this as

p Þ1 O"

The logically connective ***Þ1*** is different from ***Þ2*** in that the first connective means reliable inference where as the second connective is a plausible inference. When objects have more than one property, and/or have the possibility of having more than one property; then the situation is more complex, but still empirical in nature. This situation is addressed in the last two of Mill's canons.

The Canon of Agreement

This Canon consists of three variations. All of them begin with the same starting situation:

> *a property, p, of a class of objects { O_j } has been identified and we require evidence regarding the possible cause, c, of an object having this property.*

The *Variation for Direct Agreement* list all situations in which the property p is present. An intersection, c, is defined over the descriptions of all situations in this list.

$$c = \cap T_i$$

where { T_i } is the collection of representational sets for description of all objects O_i that are known to have the property p. In this canon we have implicit the sense that there are a set of descriptors from which in each case T_i is a subset. How these descriptors are found was not addressed in Finn's 1991 paper. However, a means to create a full set of descriptors is given in Prueitt (2001) [28] [29]. The lower case "c" is overloaded with an interpretation, and thus the notation is missing a step. The interpreted description c is of the set of descriptors $\cap T_i$.

If this intersection exist and is not empty the intersection is added to a list of meaningful "positive" descriptive components and a conjecture is made that property p is connected by a plausible relation to the descriptive intersection element c. Remember that c is a set of descriptors, perhaps composed in some way so that the resulting "sign" which is "c" is evocative of an understanding about the relationship between described properties and a category of objects.

> *If the descriptive structure c is a part of the description of the object then it is plausible that the object has property p.*

Whereas the analysis is over a class of objects { O_j } that each have a specific property p, the inference is often about whether a specific object O, not in{ O_j }, has this property. Because of the Peircean view regarding the interpretation of signs, we separate the descriptive intersection elements so that these might be made viewable through the computer user interface. This was likely not the way Finn used the results from Mill's logical canons, but is the way we now may consider.

The introduction of category theory behind the class of voting procedures, invented by Prueitt, requires some motivation. Let

$$\mathbf{O} = \{ O_1 , O_2 , . . . , O_m \}$$

be some collection of objects.

Some device is used to compute an "observation" D_r about the objects. We use the following notation to indicate this:

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[28] Prueitt P. (2001). Shallow Link analysis, Iterated scatter-gather and Parcelation (SLIP) and data visualization. Army research Office Invitational Workshop on Information Assurance, George Mason University, October 2001.

[29] In this paper, and in other related unpublished work, Prueitt used an n-gram measurement of text, scatter gather based on co-occurrence, and the development of a framework based bi-level foundation for implementing Mill's canons. This work is not complete.

$$D_r : O_i \;\text{-->}\; \{ t_1 , t_2 , . . . , t_n \}$$

This notation is read "the observation D_r of the object O_i produces the representational set

$$\{ t_1 , t_2 , . . . , t_n \}"$$

Let **P** be the union of all individual object representational sets $\mathbf{T}_k$ made during the observation of a set of objects, **O**.

$$\mathbf{T} = \cup \mathbf{T}_k.$$

This notation will be further developed in the last sections of this paper. The descriptive intersection elements are then subsets of **T**. The elements of **T** may be encoded using innovations that Prueitt has discussed elsewhere, so that voting procedures instrument each of the logical canons.

Let p be from a list of possible properties of an object O. The lower case, "c", "d', "d-c", etc; is used as above to indicate interpreted descriptive elements, composed from the elements of **T**. The elements of **T** are separated from the set of descriptive elements as a means to require an interpretant to make a composition, or induction. We assume that the truth of p has been positively assessed. As was discussed above, this assessment is stated in the form:

p Þ1 O

Which is read: "it is reliable that object O has property p."

We now interpret the *Variation for Direct Agreement* using a second logical connective, a ***partially defined relationship, Þ2*** :

c Þ2 O

This should be read: "it is plausible that a description c is related to a cause of a property similar to property p and that object O has this property. However, using some equivalence classes we get the following statement:

> *Interpreted substructure c is a* ***plausible cause*** *of property p being a property of object O.*

Again, note that the question of which property is under discussion is not explicitly stated. The expression "c Þ2 O " is "about" a single property, the identity of which is not part of expression. A separate data system is required to store information about properties. In this system, the distinction between these two logical connectives is taken into account.

So called negative knowledge played an important role on the Soviet QAT. The *Variation for Inverse Agreement* lists all situations where a property p is absent. An intersection, d, is defined over the description of all situations in this list.

$$d = \cap T_i$$

where $\{ T_i \}$ is the collection of representational sets for description of all objects O_i that are known not to have the property p. Again note the "=" will involve an interpretation and that different interpretations may be made from exactly the same set $\cap T_i$.

If an intersection, d, in the description's representational elements exist and is not empty then this intersection set is added to a list of meaningful "negative" descriptive components and a conjecture is made that property p connected by a plausible relation

d Þ2 ~ O.

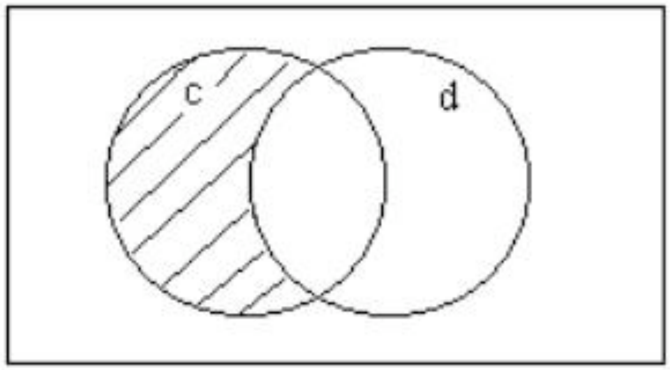

Figure 1: The representational set c - d.

The descriptive element c – d is then a disqualifier for the object O have the property under examination.

This is read, " the presence, in O, of the substructure d implies that the object O does not have the property p".

We could also interpret this to mean:

~ d Þ2 O,

but only under restricted circumstances. It is at this point that we can add various scholarships on perception and inference. But again, this is likely never been concerned as part of a bi-level cognitive aid.

Interpretations of Descriptive Elements

Let M^{+}_{p} a set of positive examples of objects having a specific property, p, and M^{-}_{p} be the class of similar objects that do not have this property.

Note that

~d Þ2 O and c Þ2 O

could imply that

c - d Þ2 O,

where c - d is set c take away the elements of set d. In this case is said to "block" some of the representational elements in c. Peirce's notion about the necessity of interpretation makes a distinction between a subset of the set of all descriptive elements and the interpreted elements c, d, c – d, ~d, etc.

The consequences of this are hard to interpret in general. Interpreted elements may be refined, and may even change over time. A study of this involves perturbations to the inference engine in the form of variations in the subsets related to reliable and plausible indicators. This variation does not change the set of descriptive elements, but does change the set of inferences.

If c is already an intersection of "positive" representational sets, then the additional removal of some elements may provide a more minimal concept structure by which to refer to a cause of the property p. However; in each case, this possibility must be tested empirically. Finn worked out the means guiding an empirically grounded testing activity. For him, the *Double Variation of Agreement* is exactly a combination of *Variation for Direct Agreement* and *Variation for Inverse Agreement*. This double variation is a method for teasing out minimal representations for the measured indicators of properties of objects.

It seems that two different possibilities exist for the *Double Variation of Agreement*. In both cases, we identify an intersection of a class of examples. One is a class of negative examples and one is a class of positive examples. In both cases, we treat the agreement as over a number of examples.

An intersection c, of representational sets, can be the basis of a conjecture about a positive cause of the property p. Likewise, the intersection d, of representational sets, can be related a conjecture about a negative cause of the property p. The subsets c – d (read, "c take away d) (see shaded area in Figure 1) and d – c can be used in some cases to refine the relationship between causes and properties. Thus three types of conjectures can be derived with the first canon.

How the classes of positive and negative examples are selected is relevant, and this selection criterion is also at the root of similar variations on the second and third Mill canons.

The Canon of Difference

For the *Canon of Difference* we again obtain descriptions of a class of situations. Certain of the objects in the situations are described as having property p. For example, again we may consider the properties as related strongly to the declarative placement of all objects into one of q categories; e.g., this object has or does not have this property.

Again, we assume that the description includes a list of representations about the composition of the objects. These descriptions are made as logical statements, such as Standard Query Languages (SQL) statements, that use representational elements from a set **T**. We may use other retrieval and search standards. The set of interpreted descriptions and the set of descriptors are encoded as different things, as are sets of possible properties and encodings of object representations.

As before, let M^{+}_{p} , be a set of positive examples of an object having a specific property, p, and M^{-}_{p} be the class of similar objects that does not have this property.

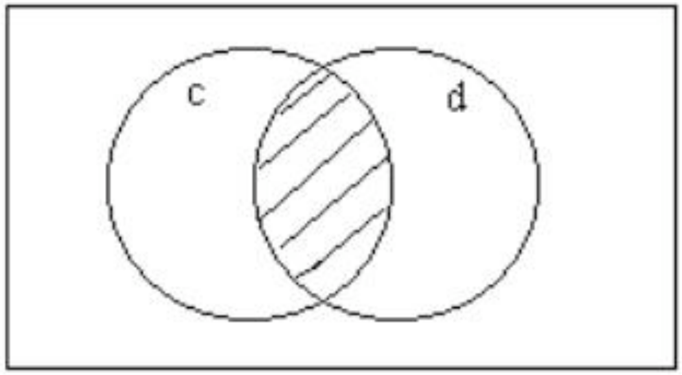

Figure 2: The intersection between representational sets c and d.

Let O_i be a single element of M^+_p and O_j be a single element of M^-_p. Let c be the interpreted representational set for O_i and d be the interpreted representational set for O_j. The intersection can be conjectured to be the descriptions of how the two objects are "entangled".

If this was the point of an interpretation, the set, c – d, is the effect, or the cause, or an indispensable part of the cause, of the property p. It may be that the, set d – c, is the effect, or the cause, or an indispensable part of the cause, of a different property q. The interpretation is what makes the inference. This interpretation is to be stored in the database. Note that the object might be a category representational set or even an intersection of some type derived from the canon of agreement, or from a series of validating steps.

Joint Canon of Agreement and Difference

The first three of Mill's logical canons were discussed in Francis Bacon's great work. Francis Bacon is regarded as the father of the scientific method.

> In his magnum opus, *Novum Organum*, or "new instrument", Francis Bacon argued that although philosophy at the time mainly used deductive syllogisms to interpret nature, mainly owing to Aristotle's logic (or Organon), the philosopher should instead proceed through inductive reasoning from fact to axiom to physical law. (Wiki reference[30])

Again, suppose we have a set of positive examples, M^+_p, of objects having a specific property, p, and a set of negative examples , M^-_p, when similar objects do not have this property.

We let C_q be the category defined by M^+_p. An intersection V^+ of the compositional representations of the positive examples M^+_p is made. The intersection V^- is defined over the set M^-_p.

We also look for one example of an object, O, that was not placed into category C_q while at the same time this object's representational set, d, has an non-empty intersection with M^+_p.

In the case we have that

$$V^+ \cap d \text{ Þ2} \sim O$$

[30] Francis Bacon wiki: http://en.wikipedia.org/wiki/Francis_Bacon

The same is done with the negative examples to produce the subset of representations M^-. One positive example is chosen and its representational set, c, used to produce a conjecture about a positive cause.

$$V^- \cap c \text{ Þ2 } O$$

The plausible inferences: $V^+ \cap d$ Þ2 ~ O and $V^- \cap c$ Þ2 O are defined as "dual formal (positive and negative) causes" of p. The use of such dual statements produces a distributed assessment of category placement.

Objects with Multiple Properties

An object O not only has the possibility of having one of several different properties, but also has the possibility of having multiple properties at the same time. The first three canons assume that only one property is being considered. The last two canons treat the more complex case.

In the general case we may formalize plausible inference regarding a substructure a_i being the reliable cause of a property, p_i. In the several advances made by Finn[31], two logical connectives are linked together, one for plausibility and one for reliability. The way in which a judgment on the strength of the inference is varied suggests that degrees of reliability and plausibility should be developed. This development may be connected to either rough sets[32] or fuzzy sets[33].

In QAT-like systems, we have three classes of logical atoms; **O** (objects), **P (**properties), and **A (**substructures.) Substructures are measured with descriptions; **T**. Certain subsets of descriptions become interpreted as the elements of substructure.

Only to the degree that it is reasonable to make an assumption of independence between the causes of properties, we can speak about residues and concomitant variation. This principle is noted in several schools of thought as a requirement that certain types of separation will be measurable in cases where several natural categories are the object of good measurement.

Suppose we have established k conjectures of the form:

$$\text{For } i = 1, \ldots k; \quad p_i \text{ Þ1 } O \quad \text{and} \quad a_i \text{ Þ2 } O$$

This maybe read, "For i = 1, . . . k, the property p_i is a reliable property of the object O and substructure a_i is the plausible cause of property p_i in object O." Under the assumption of k independent casual linkages, we can use the compact notation:

$$(p_1, \ldots, p_k) \text{ Þ1 } O \text{ and } (a_1, \ldots, a_k) \text{ Þ2 } O$$

or just,

$$(a_1, \ldots, a_k) \text{ Þ2 } O$$

in the case that the property set $(p_1, \ldots, p_k)$ has already been identified.

[31] Finn, Victor (1996b) Basic concepts of Quasi Axiomatic Theory, presented at the QAT Teleconference, New Mexico State University and the Army Research Office, December 13, 1996.
[32] Pawlak, Zdzisław (1982). "Rough sets". *International Journal of Parallel Programming* **11** (5): 341–356. doi:10.1007/BF01001956
[33] L. A. Zadeh (1965) "Fuzzy sets". *Information and Control* 8 (3) 338–353.

In the case where it is necessary to make the relationship between substructure and property explicit, then we use the notation:

$$a_i \text{ Þ2 } (O, p_i).$$

This is to read "substructure a_i is the plausible cause of the object O having the property p_i". This notation assumes that p_i Þ1 O; e.g., that the object O has property p_i is a reliable inference.

Canon of Residues

Both the Canon of Residues and the Canon of Concomitant Variation may deal with complex causes and complex properties.

The first three canons can be used to identify the meaningful subsets of the set of representational elements **T**. The last two canons are used, in our interpretation, to further delineate causal linkages between substructures and properties.

We may set aside some description from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents. There may be descriptions that do not account for inferences already taken. The residue of the phenomenon is the effect of the remaining antecedents. There is a possible matching between everything not explained and that part of our understanding that is not explaining anything. This logical canon assumes that some separation of natural category has already occurred and is present in our deductive machinery. We will illustrate.

Let **C** = { C_i } be a class of categories of objects. We assume that this class is a reasonably complete description of the similarity classes of the set of emergent wholes that are produced by a set of substructural elements (atoms). Again, reflect on the example of the atomic elements, with its periodic table, and chemistry. The level of observation of properties chemical compounds might be will separated and reliable. What is not reliable has to do with the incomplete measurement of an unknown complex molecule such as a protein, or the plausible behavior of a society under crisis. The problem addressed by Soviet QAT was how to make plausible inferences about properties of complex phenomenon such as exist in nature.

Let **A** be a generalized product of some subsets, $\{a_1 , . . . , a_q\}$, of the set **A** of substructures:

$$\mathbf{A} = (a_1 , . . . , a_q)$$

that are observed to describe a complex set of properties **P**:

$$\mathbf{P} = \{p_1 , . . . , p_r\}$$

Suppose further that r = q and we know that for each i: i= 1, 2, . . . , q-1

a_1 Þ2 (O, p_1),

a_2 Þ2 (O, p_2),

. . . ,

a_{r-1} Þ2 (O, p_{q-1}),

It is possible to use the Canon of residues to conjecture that a_r Þ2 (O, p_r).

There is a context for this conjecture. The context is the set of substructures involved in composing objects belonging to one of the categories in **C** = { C_i }. These categories in turn are part of a knowledge base build up to encode knowledge of properties of whole events, or objects. At present, this type of system is only approximated, perhaps, by the best of our automated knowledge management systems.

The Canon of Concomitant Variation

In this canon we have descriptions of the properties of two objects A and B. This is a simpler case than the pervious canon.

Linkages are conjectured. Perhaps the objects are two winter storms A and B and we are noting that two of the system observables seem to be proportionately varying. The connection is observed by differences seen in a common property. The cause of the variation in the property is conjectured to be though a specific variation in the substructure.

Define a non-specific composition function comp(.) to be a transformation of some set of substructural elements into a whole that has a set of properties. We suppose here that the properties are all functional properties of whole objects. We again suppose that structural-functional relationships have some degree of independence; i.e., that the functional properties are distinct and that, at least as a part of the whole, that distinct structural components are known to compose into distinct properties.

This is expressed:

$$\text{comp}(d + c) \sim \text{comp}(d) + \text{comp}(c)$$

where ~ is the connective "is similar to", and d , c are substructures. Of course, this is a strong assumption that is hedged by the use of the similarity connective.

Let A and B have a complex of properties:

$$(p_1 , p_2 , \ldots, p_{n-1} , \text{comp}(c)) \text{Þ1 A}$$

$$(p_{n+1} , p_{n+2} , \ldots , p_{n+m-1} , \text{comp}(d)) \text{Þ1 B}$$

and the degree of the presence of substructures c and d is ordered. We suppose that

$$c \text{ Þ2 } (A, p_{n,}),$$

and

$$d \text{ Þ2 } (B, p_{n+m}),$$

where $p_n = p_{n+m,}$ is a common property shared by object A and object B.

Let c^+ and d^+ denote an increase in c and d correspondingly and c^- and d^- denote a decrease of c and d. Since d is a substructure, d^+ and d^- maybe defined either quantitatively or qualitatively (through substructural similarity analysis.)

Then if the situation:

$$(p_1 , p_2 , \ldots, p_{n-1} , \text{comp}(c^+)) \text{Þ1 A}$$

coincides with the situation:

$$(p_{n+1} , p_{n+2} , \ldots , p_{n+m-1} , \text{comp}(d^+)) \text{Þ1 B}$$

then we can say that c and d are directly related. A similar relationship exists when comp(c^-) and comp(d^-) vary directly to produce B and A .

In the opposite case, if the situation

$$(p_1 , p_2 , \ldots, p_{n-1} , \mathrm{comp}(c^-))\ \text{Þ1}\ A$$

coincides with the situation

$$(p_{n+2} , p_{n+3} , \ldots , p_{n+m-1} , \mathrm{comp}(d^+))\ \text{Þ1}\ B$$

we say that c and d are inversely related. A similar relationship exists when comp(c^+) and comp(d^-) vary inversely.

Clearly, the above notation only begins to define the full set of possibilities for an algorithmic calculus based on Mill’s reasoning. There must be; however, some finesse in it’s application to complex problems. Mill’s logic breaks down to the degree that the set of observables, both of properties and substructures, are not composible into independent causal linkages. Moreover, natural complex systems might not be fully reducible to independent causal linkages, and a degree of skepticism is required regarding both reductionism and it’s alternatives. However, as a practical matter we do find reducibility is a useful assertion.

The problem we see is not the viability of complex descriptions of bi-level causation, but rather that these descriptions must be situational in nature. A viable situational form of extensions to Mill’s logic might be based on behavioral evidence that natural systems behave more predictably in well-defined situational context. In the case that the context has changed, we may find that the use of certain methods, depending on separation of natural categories, will fail. This failure itself is significant that that if methods are well developed we may use the failure of the system to be a indicator that the system of inference is out of context. Methods are well developed then if the system is in context and the results of our algorithms are producing good matches to observed reality.

Part Three: Situational Language and Bi-level Reasoning

Using our interpretation of QAT-like formal languages, we conjectured in 1995, that the J. S. Mill’s method creates deductive machinery that is situational in nature[34]. Acting on this conjecture, we applied a simple form of Mill’s logics to autonomous text understanding[35]. A data repository for storing information provided a framework that did not depend on specific situations. We developed a separate formalism that deals only with the "disembodied" substructure of classes of objects[36]. The methodology built a complete set of representational symbols for sufficient reference to possible semantics. This framework seems to completely implement the first three cannons, and to suggest ways in which all of the canons might be used as a means to study the behavior of complex natural phenomenon such as human discourse, or the properties of complex

[34] Prueitt, Paul S. (1995a). “A Theory of Process Compartments in Biological and Ecological Systems”, in the Proceedings of IEEE Workshop on Architectures for Semiotic Modeling and Situation Analysis in Large Complex Systems; August 27-29, Monterey, Ca, USA; Organizers: J. Albus, A. Meystel, D. Pospelov, T. Reader

[35] Prueitt, P. (1997). Quasi Axiomatic Theory, represented in the simplest form as a Voting Procedure. Presented in Moscow at a conference held at VINTI, and published in All Russian Workshop in Applied Semiotics, Moscow, Russia. (Translated into Russian and published in VINITI Conference Proceedings.)

[36] Prueitt, P. (1998). An Interpretation of the Logic of J. S. Mill, in IEEE Joint Conference on the Science and Technology of Intelligent Systems, Sept. 1998, NIST.

proteins.

The representational problem must be treated independently since the measurement of features, from which substructure is inferred, and properties is a difficult task in itself. The representational problem is not solved perfectly by any known algorithmic system. Our hope is that a certain amount of failure in representational fidelity might be compensated by adaptation within the framework. This adaptation need not be "simple" and might involve the use of evolutionary programs such as artificial neural networks or genetic algorithms. But these programs would be sub components within a framework that was essentially the Mill's logic as appears in Finn's work in quasi-axiomatics.

In situational logics the interpretation of how logical atom fit together to form inferences are specific to situational classes. The object of analysis is assumed to be in a context that maps to one of a known situational class. When the current situation cannot be mapped to the assumed situational class, then the logic must be recomputed from an elementary re-measurement of class and substructure invariants. In this case, either the representational fidelity or the logical formalism is inadequate. This means that there are two types of failures to the system we envision. The first is a failure that is easy to fix. The second involves a rather complete washing out of the old system, and the redevelopment of a new system. The unknown at this point is regarding what in the architecture remains even in the more difficult case.

The Mill's logic is naturally bi-level, and in this way set a new stage for logical analysis. Object prototypes are considered as situational classes, as are modal properties of the environment. Substructural elements are also considered prototypes, but at a distinct level of organization that is not locally meaningful to the situational classes of assembled wholes. A nesting of organizational scales is orchestrated in ways that are difficult to describe.

Two levels of organization are identified and maintained in separated data structures. There must be some "cross-organizational" scale mechanisms. We have suggested that these are involved in replication of instances of categories. The meaningful subsets of representational elements have both internal and external linkages, the discovery of which leads to one of many possible situational logics. We interpret the internal linkages to be structural in nature and the external linkages to be functional in nature.

Structural components are the cause of functional properties that result from the formation of a whole that is greater than the sum of the structural components. Water from hydrogen and oxygen is an example. The compound, water, does not depend on having specific examples of an oxygen atom, but rather any one of a class of atoms that is the prototype class for all oxygen atoms.

Extension of Notation

We will follow and further develop a notation introduced in discussions of bi-level voting procedures, as seen last two sections of this paper.

However, the objects will be generalized from text passages to generic objects. Categorization policies are generalized to similarity classes. We are interested in the property that a "description" is the "formal cause" of an object being placed into a similarity class. This is clearly a "synthetic" property that is to be defined by careful empirical methods and by forming good representations of objects.

The introduction of the category theory behind the class of voting procedures requires some motivation. Let

$$\mathbf{O} = \{ O_1 , O_2 , . . . , O_m \}$$

be some collection of objects.

Some device is used to compute an "observation" D_r about the objects. We use the following notation to indicate this:

$$D_r : O_i \;\text{-->}\; \{ t_1 , t_2 , . . . , t_n \}$$

This notation is read "the observation D_r of the object O_i produces the representational set

$$\{ t_1 , t_2 , . . . , t_n \}"$$

We now combine these object level representations to form a category representation.

· each "observation", D_r, of the objects in the training set **O** has a representational set

$$D_r : O_i \;\text{-->}\; \mathbf{T}_k = \{ t_1 , t_2 , . . . , t_n \}$$

· Let **P** be the union of all individual object representational sets $\mathbf{T}_k$.

$$\mathbf{P} = \cup\, \mathbf{T}_k.$$

This set **P** is the representation set for the complete collection $\mathbf{O_1}$.

· The set **P** can be partitioned, with overlaps, to match the assignment of objects to categories **C** = { C_q }. Let $\mathbf{T^*_q}$ be the union of all elements of the representation sets $\mathbf{T}_k$ for all objects that are assigned to C_q.

$$\mathbf{T^*_q} = \cap \{ \mathbf{T}_k \mid \text{object } O_k, \text{ is assigned to category } C_q. \}$$

In this way, the category representation set, $\mathbf{T^*_q}$, is defined for each category C_q.

The overlap between category representation $\mathbf{T^*_q}$, and $\mathbf{T^*_s}$, is one statistical measure of the "entanglement" between categories C_q and C_s. This fact leads to a method for identifying the minimal intersections of descriptions of structural features from the category representational sets and matching these minimal intersections to logical atoms in quasi axiomatic theory.

The use of voting procedures to apply the first three cannons is straightforward, only needing to be tested computationally using some application such as cyber security or text based security analysis.

On Lattices

We now introduce some additional mathematical constructions that might be used in QAT-like systems to keep books on the set of all subsets of the representational elements used in descriptions. These subsets are nodes of the lattice of subsets with smallest element the empty set and largest element the set of all representational elements, the universal set, from a class of descriptions.

The notion of minimal meaningful intersections can be seen using a picture of the lattice. In Figure 3 we see some representational sets and some subsets. The nodes of the lattice stand for subsets, arranged by the partial relationship "set inclusion". The nodes form a large diamond shape with the universal set at the top and the empty set at the bottom.

Note that set inclusion is not a total order since, for example T_1 and T_2 are not ordered by this relationship. In the figure, the node m_1 could be the intersection of T_1 and T_2 and m_2 could be the intersection of T_1 , T_2 and T_i

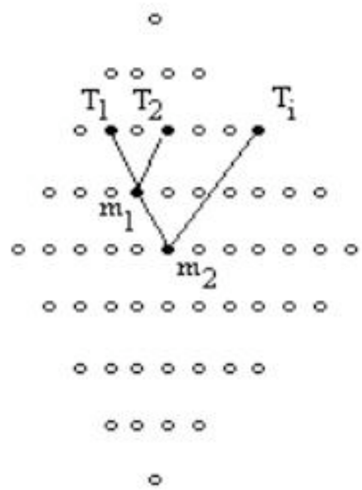

Figure 3: Some substructures and relationships in the lattice of all subsets of the set of all representational elements.

Note that if some manageable set of lattice nodes are identified as having properties and internode relationships then we have some of the constructions seen in semantic nets. These constructions have the syntagmatic form < a, r, b > where a and b are locations and r is a relational property.

It is also worth noting that the size of the lattice is the number 2 to the power of the size of the universal set. In text understanding systems the universal set can be many thousands of elements. Thus the lattice is very large indeed. However all intersections of passage (object) representational sets will be in a relatively small part of the bottom of the lattice.

Description of the Minimal Voting Procedure (MVP)

To instantiate a voting procedure, we need the following triple $< C, O_1, O_2 >$:

A set of categories $C = \{ C_q \}$ as defined by a training set O_1.

A means to produce a document representational set for members of O_1.

A means to produce a document representational set for members of a test set, O_2.

We assume that we have a training collection O_1 with m document passages,

$$O_1 = \{ d_1 , d_2 , . . . , d_m \}$$

Documents that are not single passages can be substituted here. The notion introduced above can be generalized to replace documents with a more abstract notion of an "object".

Objects

$$O = \{ O_1 , O_2 , . . . , O_m \}$$

can be documents, semantic passages that are discontinuously expressed in the text of documents, or other classes of objects, such as electromagnetic events, or the coefficients of spectral transforms.

Some representational procedure is used to compute an "observation" D_r about the semantics of the passages. The subscript r is used to remind us that various types of observations are possible and that each of these may result in a different representational set. For linguistic analysis, each observation produces a set of theme phrases. We use the following notion to indicate this:

$$D_r : \text{-->} \{ t_1 , t_2 , . . . , t_n \}$$

This notion is read "the observation D_r of the passage di produces the representational set $\{ t_1 , t_2 , . . . , t_n \}$"

We now combine these passage level representations to form a category representation. Each "observation", D_r , of the passages in the training set O_1 has a "set" of theme phrases

$$D_r : \text{-->} T_k = \{ t_1 , t_2 , . . . , t_n\}$$

Let A be the union of the individual passage representational sets T_k.

$$A = \text{Union } T_k.$$

This set A is the representation set for the complete training collection O_1 .

The set A can be partitioned, with overlaps, to match the categories to which the passages were assigned. Let T^*_q be the union of all theme phrase representation sets T_k for all passages that are assigned to the category q.

$$T^*_q = \text{ Union } T_k \text{ such that, } d_k\text{, is assigned to the category q.}$$

The category representation set, T^*_q, is defined for each category number q.

The overlap between category representation T^*_q, and T^*_s, is one statistical measure of the "cognitive entanglement" between category q and category s. This fact leads to a method for identifying the minimal intersections of structural features of the category representational sets.

J. S. Mill’s logics relies on the discovery of meaningful subsets of representational elements. The first principles of J S Mill’s argumentation are:

1. that negative evidence should be acquired as well as positive evidence

2. that a bi-level argumentation should involve a decomposition of passages and categories into a set of representational phrases

3. that the comparison of passage and category representation should generalize (provide the grounding for computational induction) from the training set to the test set .

It is assumed that each "observation", D_k, of the test set O_2 is composed from a "set" of basic elements, in this case the theme phrases in A. Subsets of the set are composed, or aggregated, into wholes that are meaningful in a context that depends only statistically on the characteristics of basic elements.

This general framework provides for situational reasoning and computational argumentation about natural systems.

For the time being, it is assumed that the set of basic elements is the full phrase representational set

$$A = \text{Union } T_k.$$

for the training collection O_1. Given the data:

$$T^*_q \text{ for each } C_q, q = 1, \ldots, n$$

and the representational sets T_k, from the observations D_k, for each passage, d_k, from the test set O_2, we generate the hypothesis that the observation D_k should be categorized into category q.

This hypothesis will be voted on by using each phrase in the representational set for D_k by making the following inquiries for each element ti of the representational set T_k:

1. does an observation of a passage, D_k, have the property p, where p is the property that this specific representational element, ti , ***is*** also a member of the representational set T^*_q for category q.

2. does an observation of a passage, D_k, have the property p, where p is the property that this specific representational element, ti , ***is not*** a member of the representational set T^*_q for category q.

Truth of the first inquiry produces a positive vote, from the single passage level representational element, that the passage is in the category.

Truth of the second inquiry produces a negative vote, from the single representational element, that the passage is not in the category. These votes are tallied.

Data Structure for Recording Votes

For each passage, d_k , we define the matrix A_k as a rectangular matrix of size m x h where m is the size of a specific passage representational set T_k, and h is the number of categories. The passages are indexed by k, each passage has it's own matrix.

Each element t_i of T_k, will get to vote for or against the hypothesis that this kth passage should be in the category having the category representational set T^*_q. Thus A_k is defined by the rule:

$$a_{i,j} = -1 \text{ if the phrase is not in } T^*_q$$

or

$$a_{i,j} = 1 \text{ if the phrase is in } T^*_q$$

Matrix A_k is used to store the individual + - votes placed by each agent (i.e., the representational element of the phrase representation of the passage.)

This linear model produces ties for first place, and places a semi-order (having ties for places) on the categories by counting discrete votes for and against the hypothesis that the document is in that category.

Data Structure to Record Weighted Votes

A non-linear (weighted) model uses internal and external weighting to reduce the probability of ties to near zero and to account for structural relationships between themes.

Matrix B_k is defined:

$$b_{i,j} = a_{i,j} * \text{weight of the phrase in } T_k$$

if the phrase is not in T^*_q or

$$b_{i,j} = a_{i,j} * \text{weight of the phrase in } T^*_q$$

if the phrase is in T^*_q

This difference between the two multipliers is necessary and sufficient to break ties resulting from the linear model (matrix A_k).

For each passage representation and each category, the tally is made from the matrix Bk and stored in a matrix C having the same number of records as the size of the document collection, and having h columns – one column for each category.

The information in matrix C is transformed into a matrix D having the same dimension as C. The elements of each row in C are reordered by the tally values. To illustrate, suppose we have only 4 categories and passage 1 tallies {-1214,-835,451,1242} for categories 1, 2, 3 and 4 respectively. So

cat1 --> -1214, cat2 --> -835, cat3 --> 451 and cat4 --> 1242.

By holding these assignments constant and ordering the elements by size of tally we have the permutation of the ordering (1, 2, 3, 4) to the ordering (4, 2, 3, 1).

(1, 2, 3, 4) --> (4, 2, 3, 1).

This results show that for passage 1, the first place placement is category 4, the second place is category 2, etc. The matrix D would then have (4, 2, 3, 1), as its first row.

Appendix A: Discrete Homology to Axiomatic Systems

Research Note: December 20, 2011

Paul Stephen Prueitt, PhD

Abstract: A formal definition of homology between a set of discrete state transitions and a trajectory in n-dimensions is discussed in the context of models of learning in biological systems. Logical and physical entailment might then be mirrored.

The simplest form for a Lie group[37] may be seen as an algebraic model of the behavior of a set of linear transformations, for example as used in modeling visional flow[38]. A Lie group is something that is simultaneously an algebraic group and a manifold. A good example of a Lie group is a set of matrices defined as continuous transformation on the points of a vector space. The group properties include a closure property, and associate property, the existence of an identity element and inverses.

The concept of a discrete and finite Lie group is a difficult one and may be seen as "unnatural". For example, the inverse of an operation that moves a point s(j) to point s(i) might be equated with a reach ability argument that a composition of steps starting at s(j) will eventually, in a finite number of steps reach back to s(i). This requires that all transition be part of a sequence that returns to previous states. We may require that all state transitions, { t(k) } be part of a cycle; e.g., if t(k)[s(i)] → s(j), there must exist a finite sequence of state transactions that compose to bring s(j) back to s(i). However, this may not be enough to satisfy the definition of an algebraic group.

Such compositions require an associative law. It is not clear how this may be defined. Closure also requires some abstraction since for state transition diagrams; state transitions are defined only on one state. However, this problem is connected with the vast difference between a discrete topology[39] and a topology similar to the topology of open sets in the real line. It is supposed therefore that any notion of a discrete Lie Group must be defined as a construction having certain homological properties with a Lie Group defined on a non-discrete space, in which the discrete space is embedded. It is this notion of a "matching" between a finite state transition diagram and a Lie group that we are concerned with.

For our purposes here, it is proper to consider only those transformations that take a location within an n- dimension manifold to a, possibly, different location in the same manifold. The manifold may be defined by a set of first order ordinary differential equations. A general question arises, might a discrete group be defined that encodes any finite state transition diagram, including quasi-axiomatic logics[40]; e.g., as in a Mill's logical cannon derived logical entailment systems[41]. This problem is not fully resolved. The current paper is designed to identify areas where incomplete formal work exists.

The question of homology; e.g., reliable mapping between logical and physical entailments, is then a question of mapping physical entailments to logical entailment, and visa versa. In logical

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37 B. C. Hall. Lie Groups, Lie Algebras, and Representations: An Elementary Introduction. Springer, 2003

38 E. Bayro-Corrochano and J. Ortegon-Aguilar. Lie algebra approach for tracking and 3d motion estimation. Image and Vision Computing, 25:907–921, 2007

39 Discrete Topology http://en.wikipedia.org/wiki/Discrete_topology

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systems we may see the single step logic as single steps along a path, or trajectory, created by the transformations from n dimensions to n dimensions. These discrete logical paths; e.g., logical entailment, are generally defined using a transition state rule; e.g.,

$$(1, 0 , 1 0) \rightarrow (0, 0, 0, 1) \rightarrow (1, 1, 1, 1),$$

Both logical rules and the dynamical rules have domain and range as subsets of the n-dimensional manifold. If we consider the abstract properties of transition state diagrams we may find that these diagrams encode all necessary dynamic entailment necessary to define models of biological functions. However, the central question is regarding if an arbitrary finite state diagram may be extended to a finite state diagram having sufficient properties to be embedded as a homology to a class of simple Lie Algebras. This question is not closed.

One such example is the set of generalized immunological response transition diagrams[42]. The transition diagrams in Eisenfeld and Prueitt (1988) shows a complete discrete model of high and low zone tolerance response behaviors characterizing any immunological response to novel or recognized antigens. A system of piece wise defined first order differential equations was shown by Prueitt (1988) to pass through all appropriate regions of the associated n dimensional manifold. This was an original contribution of Prueitt, which is extended in various later papers[43 44 45 46 47].

A formal process for encoding axiomatic systems as finite state transformations having certain algebraic closure and associative rule, is developed in additional publications[48 49]. The basic definitions of a discrete to continuum homology are defined in Prueitt's PhD thesis (1988)[50]. The idea then, as it is now, is to create an ability to encode in real time wave interactions any behavior of a continuum manifold, e.g., one that arises in the presence of a system of first order differential equations. Discrete to continuum manifold mapping, as was shown in the case of generalized immune response in Prueitt's thesis, suggests the possibility of electro magnetic wave interference patterns to compute a discrete logic reliably[51]. A computational model might be

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developed that provides additional evidence that cognitive processes are supported by dendrite-to-dendrite interactions in neuronal groups[52].

The homology theory shown in Prueitt's PhD thesis can be generalized easily to stochastic equations, so that the categorization of measurement may be discretized. The discretization is not merely to a logical value but also to a normally distributed random variable. A bursting model of neural associative interactions, seen in [53], and widely known; is then modeled by the input caused movement in the logical or the continuum space. The Pribram neuro-wave model[54] governing field-to-field interactions between communities of neurons is also present. The control of groups of neurons by a single parameter is discussed by several of Pribram's colleagues[55]. Edelman discusses neuronal group-selection[56]. Field to field processing is the basis for the contribution made by Pribram; e.g., his theory of holonomic brain processing.

Incomplete and/or uncertain information is a big deal. The incomplete knowledge of situation may be input into these homologies with some but not all of the state values set to zero. The system should produce an output (consequence of physical entailment) that guesses at a classification category, as if "normal" presence of hidden information were input along with the non zero inputs. Those guesses that are judged to be correct may be used to modify the underlying n-dimensional continuum manifold, and thus a utility function may govern the evolution of a real inference engine. In other work[57], we will address the question of information that is judged to be first occurrences of something; e.g., as not conceivable by the cognitive system. In this case, the overall biological response is to turn the matter over to an immune system's interface with cognitive and memory systems.

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